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How many positive integers  $n$  are there such that  $n$  is a multiple of 5, and the least common multiple of  $5!$  and  $n$  equals 5 times the greatest common divisor of  $10!$  and  $n$ ?

- (A) 12    (B) 24    (C) 36    (D) 48    (E) 72

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Let  $N$  be the number of consecutive 0's at the right end of the decimal representation of the product  $1!2!3!4! \cdots 99!100!$ . Find the remainder when  $N$  is divided by 1000.

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Let  $P(x)$  be a polynomial with rational coefficients such that when  $P(x)$  is divided by the polynomial  $x^2 + x + 1$ , the remainder is  $x + 2$ , and when  $P(x)$  is divided by the polynomial  $x^2 + 1$ , the remainder is  $2x + 1$ . There is a unique polynomial of least degree with these two properties. What is the sum of the squares of the coefficients of that polynomial?

- (A) 10    (B) 13    (C) 19    (D) 20    (E) 23

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Real numbers  $x$  and  $y$  are chosen independently and uniformly at random from the interval  $(0, 1)$ . What is the probability that  $\lfloor \log_2 x \rfloor = \lfloor \log_2 y \rfloor$ ?

- (A)  $\frac{1}{8}$     (B)  $\frac{1}{6}$     (C)  $\frac{1}{4}$     (D)  $\frac{1}{3}$     (E)  $\frac{1}{2}$

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How many nonzero complex numbers  $z$  have the property that  $0$ ,  $z$ , and  $z^3$ , when represented by points in the complex plane, are the three distinct vertices of an equilateral triangle?

- (A) 0    (B) 1    (C) 2    (D) 4    (E) infinitely many